A universal method for quantitative analysis of triboelectric nanogenerators†

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Triboelectric nanogenerators (TENGs) are promising materials for harvesting mechanical energy from the environment. Although different structures and modes of TENGs have been designed in recent years, a theoretical model that can comprehensively describe all kinds of TENGs has not been developed yet. Therefore, in this work, a universal edge approximation based equivalent capacitance (EDAEC) method is used to demonstrate charge distributions and the electric field in TENGs, providing quantitative analysis for all modes of TENGs. Analytical models and the quantitative Q–V–x relationship are built for various TENGs, including the contact-separation (CS) mode, contact freestanding-triboelectric-layer (CFT) mode, single-electrode contact (SEC) mode, lateral sliding (LS) mode, sliding freestanding-triboelectric-layer (SFT) mode and single-electrode sliding (SES) mode. Furthermore, simulated results based on the finite element method are obtained to validate the results derived from the analytical models, which are in good agreement. This universal method for all modes of TENGs is a milestone for understanding the working principles of TENGs in-depth, optimizing TENGs’ outputs, and revealing the relationships between different modes. This work presents a theoretical basis to guide the further development of TENGs in terms of structural design toward efficient energy harvesting.

Introduction

Energy is more and more important for the development of economy and society.1,2 However, since fossil fuels are limited, the energy crisis becomes a serious challenge to the modern society. The challenge of sustainable development calls for renewable energy sources, including solar, wind, bio-fuel and tidal energy.3–7 Meanwhile, the Internet of Things (IoT) are in the spot light, which contain millions of sensors to be powered. These numerous sensors are widely distributed, which makes monitoring and exchanging their power sources an impossible task. Triboelectric nanogenerators (TENGs) were invented to harvest the mechanical energy from the environment and convert it into electricity to power sensors or portable electronic devices, which is a promising approach toward the development of the IoT.8–11

TENGs have been rapidly developed in recent years with four basic modes, including vertical contact-separation (CS) mode, lateral sliding (LS) mode, single electrode (SE) mode and freestanding triboelectric-layer (FT) mode.12 For any emerging technology, setting an analytical model is a crucial step for understanding and applying it in mass production in industry.13,14 Therefore, building analytical theoretical models to evaluate the output performance becomes more and more important for the commercialization and industrialization of the TENG technology.15 Although Niu et al. have already given the formula to describe different TENGs’ outputs,16–18 they are mostly based on traditional parallel-board capacitance models, which ignore the edge effect of the capacitance, which may make a significant impact, especially in sliding TENGs. For example, the transferred charge and the open-circuit voltage of an SFT TENG do not have a linear relationship with displacement due to the edge effect. Meanwhile, the structure of TENGs will become more and more complicated, and thus establishing theoretical models for these complicated TENGs will become a tricky but important task. Therefore, a universal method, which can give the analytical formulae (of the Q–V–x relationship) for all kinds of TENGs, becomes more and more significant.

According to the motion types and the structures, TENGs could be categorized as 6 structures, including the vertical contact-separation (CS), the lateral sliding (LS), the contact freestanding triboelectric-layer (CFT), the sliding freestanding triboelectric-layer (SFT), the single electrode contact (SEC), and the single electrode sliding (SES), as shown in Fig. 1. For contact-separation TENGs (CS, CFT, and SEC modes), researchers have already taken the capacitance of the side effect into consideration by using the formula considering the edge effects (eqn (S1) in the ESI†)19,20 to achieve a more accurate
model. However, for sliding TENGs, it is difficult for researchers to calculate their real capacitances with displacement $x$. The challenge is how to analytically describe the impact caused by the edge effects. No analytical formula has been given to describe the relationships of the transferred charge or the open-circuit voltage with the displacement, especially for the SFT and SES modes. It is vital to build models for these sliding TENGs and establish a universal method that can develop models for all kinds of TENGs.

Therefore, a universal method for quantitative analysis of all modes of TENGs is developed in this work, as shown in Fig. 1. This method is based on the edge approximation based equivalent capacitance (EDAEC). The equivalent capacitance models are used to demonstrate charge distributions on each electrode. Due to contact electrification, static triboelectric charges will be dispersed on the dielectric surface after coming into contact with metal electrodes. The triboelectric charge density of the dielectric surface is defined as $\sigma$, and then the total charge is $\sigma wL$ (the length and width of the dielectric layer are $L$ and $w$). According to charge conservation, the metal electrodes would have the same amount of opposite-sign charges in total.21,22 By defining the charges distributed on electrode 1 and 2 as $Q_1$ and $Q_2$, respectively, the relation can be given as below:

$$Q_1 + Q_2 = \sigma wL \quad (1)$$

Under short-circuit conditions, two electrodes would have the same potential. For simplicity, the two electrodes and the dielectric surface can be defined as node 1, 2 and surface $a$, respectively, as shown in Fig. 1. Therefore, the following equations can be obtained.

$$V_1 = \frac{Q_1}{C_{a1,\text{total}}} = V_2 = \frac{Q_2}{C_{a2,\text{total}}} \quad (2)$$

Thus, the short-circuit equilibrium charges $Q_1$ and $Q_2$ on electrodes 1 and 2, respectively, are given as:16

$$\begin{align*}
Q_1 &= \frac{1}{1 + \frac{C_{a2,\text{total}}}{C_{a1,\text{total}}}} \sigma wL \quad (3a) \\
Q_2 &= \frac{1}{1 + \frac{C_{a1,\text{total}}}{C_{a2,\text{total}}}} \sigma wL \quad (3b)
\end{align*}$$

From the equations above, the working mechanism of TENGs can be easily illustrated. When the distance between surface $a$ and electrode 2 is zero the capacitance across them would be much larger than that across electrode 1 ($C_{a1,\text{total}} \gg C_{a2,\text{total}}$). Most of the positive tribo-charges would be attracted to electrode 2. $Q_2$ is close to $\sigma wL$ and $Q_1$ is approximately zero. On the other hand, when the distance is quite large, the capacitance across surface $a$ and electrode 2 would be much smaller than that across electrode 1 instead ($C_{a1,\text{total}} \gg C_{a2,\text{total}}$). So $Q_1$ is close to $\sigma wL$ and $Q_2$ is approximately zero. Meanwhile, the equivalent capacitances of all kinds of TENGs are shown in Fig. 1. Thus, according to this EDAEC method the charge distributed on each electrode could then be quantitatively calculated. Because side-effect capacitance of contact-separation TENGs can be calculated from eqn (S1), the EDAEC method can also be used to calculate the output performance of these contact-separation TENGs, directly through eqn (S1). Since the analytical equations for contact-

![Fig. 1](https://i.imgur.com/1Q5Q5Q5.png)

**Fig. 1** Schematic illustration of the universal method for different kinds of TENGs. The capacitances caused by the side effect are in red.
separation TENGs have been given before, models for sliding TENGs are derived below.

**LS mode**

Niu et al. have already given the formula to describe LS-TENGs’ outputs ($Q$, $V$ and $I$) with the relationship with displacement $x$, in which the edge effect of the capacitance is ignored.\textsuperscript{17,24,25} Traditionally, by assuming the parallel-board capacitance model, the total capacitance $C_{\text{total}}$ fully depends on the capacitance of the covered part between electrodes 1 and 2, as described in Fig. 2a. The length $L$ and the width $w$ of the electrodes and dielectric layers are all defined as 100 mm. The dielectric layer with a thickness of $d_0$ is stacked on electrode 1 and the gap between the dielectric layer and sliding electrode 2 is defined as $d$. The capacitance $C_{\text{total}}$, which is based on the traditional unlimitedly large plane, can be described with the equation below:

$$C_{\text{total}} = \varepsilon_0 \frac{(L - x)w}{d_0} \left( \frac{L}{\varepsilon_0 x} + d \right)$$  \hspace{1cm} (4)

And the equivalent capacitance of $C_{\text{total}}$ can also be described as the series connection of $C_{b1(x)}$ and $C_{b2(x)}$ in which:

$$C_{b1(x)} = \varepsilon_0 \frac{(L - x)w}{d_0} \left( \frac{L}{\varepsilon_0 x} + d \right)$$ \hspace{1cm} (5a)

$$C_{b2(x)} = \varepsilon_0 \frac{(L - x)w}{d}$$ \hspace{1cm} (5b)

Here, a new analytical EDAEC model is built, taking the capacitance of the edge effect into consideration, to quantitatively describe TENGs’ outputs. The real equivalent capacitance of $C_{\text{total}}$ is shown in Fig. 2b, which includes three parts. Part 1 shows the edge capacitance $C_{a2(x)}$ between surface a (dielectric layer) and electrode 2, and the capacitance $C_{a1(x)}$ between surface a and electrode 1. According to Fig. S1,\textsuperscript{†} $C_{a2(x)}$ which is almost a constant, is quite small when compared to $C_{a1(x)}$. Therefore $C_{a2(x)}$ can be approximated as a small constant through:

$$C_{a1(x)} = \frac{\varepsilon_0 \varepsilon_0 x w}{d_0}$$ \hspace{1cm} (6a)

$$C_{a2(x)} = C_a$$ \hspace{1cm} (6b)

Part 2 in Fig. 2b shows the edge capacitance $C_{12d}$ between electrode 1 and electrode 2, which is also considered as a small constant for easy calculation. And Part 3 shows the capacitance $C_{1b2}$ based on the traditional parallel-board capacitance model between the covered parts of electrode 1 and electrode 2.

Therefore, the total capacitance $C_{\text{total,12}}$ can be calculated as:

$$C_{\text{total}} = C_{12a} + C_{12d} + C_{1b2} = \frac{\varepsilon_0 L_x w}{\varepsilon_0 x w + C_{a} d_0} + C_{12d} + \frac{\varepsilon_0 (L - x) w}{\varepsilon_0 d + d_0}$$ \hspace{1cm} (7)

With triboelectric charge density $\sigma$, the total triboelectric charge on surface sections a and b can be defined as the equations below.

$$Q_a = \sigma w(x)$$ \hspace{1cm} (8a)

$$Q_b = \sigma w(L - x)$$ \hspace{1cm} (8b)

Under short-circuit (SC) conditions, electrode 1 and electrode 2 will have the same electrical potential. Therefore, the
total charge on each electrode can be derived from eqn (3) by
overlapping electrostatically induced charge from \( Q_a \) and \( Q_b \):

\[
\begin{align*}
q_{a1}(x) &= \frac{C_{a1} + C_{a2}}{C_{a1} + C_{a2}} Q_a = \frac{\varepsilon_r \varepsilon_0 W}{\varepsilon_r \varepsilon_0 W + d_0 C_a} \sigma_{wx} \\
q_{a2}(x) &= \frac{C_{a2}}{C_{a1} + C_{a2}} Q_a = \frac{d_0 C_a}{\varepsilon_r \varepsilon_0 W + d_0 C_a} \sigma_{wx} \\
q_{b1}(x) &= \frac{C_{b1} + C_{b2}}{C_{b1} + C_{b2}} Q_b = \frac{\varepsilon_r d}{\varepsilon_r d + d_0} \sigma_w (L - x) \\
q_{b2}(x) &= \frac{C_{b2}}{C_{b1} + C_{b2}} Q_b = \frac{d_0}{\varepsilon_r d + d_0} \sigma_w (L - x)
\end{align*}
\] (9a) (9b) (9c) (9d)

Therefore, the total charge on electrode 1, \( q_1 \), can be calculated as:

\[
q_1(x) = q_{a1} + q_{b1} = \frac{\varepsilon_r \varepsilon_0 W}{\varepsilon_r \varepsilon_0 W + d_0 C_a} \sigma_{wx} + \frac{\varepsilon_r d}{\varepsilon_r d + d_0} \sigma_w (L - x)
\]

\[
= \left( \frac{\sigma_w - d_0}{\varepsilon_r d + d_0} \right) x + \frac{d_0 C_a}{\varepsilon_r \varepsilon_0 W} \sigma_w + \sigma_w L \frac{\varepsilon_r d}{\varepsilon_r d + d_0} - \frac{d_0 C_a}{\varepsilon_r \varepsilon_0 W} \sigma_w
\]

\[
= \left( \frac{\sigma_w - d_0}{\varepsilon_r d + d_0} \right) x + \frac{d_0 C_a}{\varepsilon_r \varepsilon_0 W} \sigma_w + \sigma_w L \frac{\varepsilon_r d}{\varepsilon_r d + d_0} - \frac{d_0 C_a}{\varepsilon_r \varepsilon_0 W} \sigma_w
\]

\[
= \left( \frac{\sigma_w - d_0}{\varepsilon_r d + d_0} \right) x + \frac{d_0 C_a}{\varepsilon_r \varepsilon_0 W} \sigma_w + \sigma_w L \frac{\varepsilon_r d}{\varepsilon_r d + d_0} - \frac{d_0 C_a}{\varepsilon_r \varepsilon_0 W} \sigma_w
\]

(10)

With \( x = 0 \) being assigned to the initial state, the initial \( q_1 \) is
defined as:

\[
q_{1(0)} = \sigma_w L \frac{\varepsilon_r d}{\varepsilon_r d + d_0}
\]

(11)

The transferred charge \( Q_{sc} \) is equal to the change of the
charge on electrode 1, and therefore the transferred charge \( Q_{sc} \)
can be calculated as the difference between the induced charge
\( q_{1(x)} \) and the initial state charge \( q_{1(0)} \) on electrode 1. The
short-circuit charge \( Q_{sc} \) can be calculated using the equation below.

\[
Q_{sc(x)} = |q_{1(x)} - q_{1(0)}| = \left( \frac{\sigma_w - d_0}{\varepsilon_r d + d_0} \right) x + \frac{d_0 C_a}{\varepsilon_r \varepsilon_0 W} \sigma_w \frac{Z_a^2}{Z_a} - \sigma_w Z_a
\]

(12)

In the equation above, \( Z_a \) is a constant when the structure is
fixed:

\[
Z_a = \frac{C_a d_0}{\varepsilon_r \varepsilon_0 W}
\]

(13)

It should be noticed that the first item of \( Q_{sc} \) has a linear
relationship with displacement \( x \), representing the part of \( Q_{sc,tra} = \sigma_{wx} \) from the traditional method. The second item of \( Q_{sc} \)
is a non-linear item, and the third item is a constant.

Traditionally, the graph of \( Q_{sc,tra} \) has a strictly linear rela-
tionship with \( x \). However, in the new EDAEC method, as we
consider the edge effect, \( Q_{sc} \) does not have a strictly linear
relationship with displacement. As shown in Fig. 2c, the graph
of \( Q_{sc} \) has a flatter start at the beginning (Fig. 2d), and then
the slope of \( Q_{sc} \) increases until it has almost the same slope with the
traditional graph when \( d = 0 \) (Fig. 2e). In order to validate the
above theoretical derivation, an LS mode TENG model is
simulated by the finite element method (using the COMSOL
Multiphysics software package), and the corresponding results
are plotted in Fig. 2c-e. It should be emphasized that the
theoretically calculated \( Q_{sc} \) by COMSOL simulation is more
close to that from our EDAEC model than \( Q_{sc,tra} \). According to
the results of \( Q_{sc} \), the graph of current and displacement is also
plotted with a constant sliding speed of 0.01 m s\(^{-1}\). Unlike the
traditional graph which gives constant current, the EDAEC
method shows that the current would be much lower when \( x \) is
small and then would sharply increase to the traditional value
(Fig. 2f). These results demonstrate that our EDAEC model
which considers the edge effect is more suitable when
describing LS mode TENGs.

With the total capacitance \( C_{total} \) and the short-circuit charge
\( Q_{sc} \) known, the open-circuit voltage can be obtained using
the following equation (details are shown in eqn (52)):

\[
V_{oc(x)} = \frac{Q_{sc(x)}}{C_{total}}
\]

(14)

The capacitance calculated by the EDAEC method is
demonstrated to be more accurate than traditional values
(Fig. 3a) when compared to the COMSOL results. The
open-circuit voltages are also plotted in Fig. 3b, which gradually
increase with displacement \( x \), and increase sharply when \( x \) is
quite close to \( L \). The total capacitance of an LS TENG will be zero
according to the traditional method at \( x = L \). At this position,
the open-circuit voltage will be infinitely large, which is
different from the real situation. The EDAEC method takes the
dge capacitance into consideration, and thus the open-circuit
voltage can be accurately calculated even when \( x = L \).
Compared to traditional calculation, the EDAEC method shows
that the voltage will give a result which is more consistent with
that obtained by COMSOL, as shown in Fig. 3b.

According to eqn (12), the transferred charge \( Q_{sc} \) is also
affected by the TENG’s structural parameters, for example the
distance \( d \) between the dielectric material and electrode 2, and
the thickness \( d_0 \) of the dielectric material. Therefore, the short-
circuit transferred charges and open-circuit voltages with different
gap \( d \) values are also shown in Fig. 3c and d. It is obvious
that the slopes of \( Q_{sc} \) and \( V_{oc} \) against the change of \( x \)
become much smaller when the gap \( d \) increases. This result can
explain the experimental phenomenon why the outputs (\( Q_{sc} \) and
\( V_{oc} \)) of the LS TENG become quite low when the gap between the
dielectric material and electrode 2 is larger than a threshold
value. The thickness \( d_0 \) of the dielectric material will also affect
the LS TENG’s outputs, as shown in Fig. 3e and f. The \( Q_{sc} \)
against \( x \) will become a bit smaller when the dielectric thickness
\( d_0 \) increases, while the \( V_{oc} \) will become much larger due to the
decrease of the total capacitance. These results are also
compared with the simulation results obtained using COMSOL,
which show a high degree of agreement. Based on the above
analysis, the EDAEC method, by considering the edge effect,
shows high accuracy in calculating the capacitance \( C_{total} \), short-
circuit charge \( Q_{sc} \) and open-circuit voltage \( V_{oc} \) of LS mode
TENGs under different conditions. It should be emphasized
that this EDAEC method is quite important for gratin
structured TENGs. Because the electrode sizes of grating structured TENGs are much smaller than those of other sliding TENGs, the electrodes could not be regarded as the traditional unlimitedly large plane anymore. Therefore, the influence of the side-effect capacitance will become much significant. The short-circuit charge of grating structured TENGs is no longer a straight line.\textsuperscript{26}

**Sliding FT mode**

The capacitance caused by the edge effect in sliding FT (SFT) TENGs is significant and has a much more obvious influence than that for LS TENGs. Due to the impact by the edge effect, it is obvious that the short-circuit transferred charge and the open-circuit voltage have no linear relationship with displacement, and no analytical model has been developed for SFT TENGs yet.\textsuperscript{16,27}

The simplest structure of an SFT TENG, shown in Fig. 4, consists of two adjacent metal electrodes (length: \(L = 100\) mm, width: \(w = 100\) mm) with a gap \(g\) and a movable dielectric layer with an identical size to each electrode. The dielectric layer is placed above the electrodes with a distance of \(d\). Due to the impact of the edge capacitance, the sliding displacement of the TENG can be divided into three sections according to \(x\), \(0 < x < g\) (Part 1, Fig. 4a and b), \(g \leq x \leq L\) (Part 2, Fig. 4c and d), and \(L < x < L + g\) (Part 3), which is symmetric to Part 1. Therefore, both the short-circuit transferred charge \(Q_{sc}\) (Fig. 4e and f) and open-circuit voltage (Fig. 4g and h) can be divided into 3 parts correspondingly: the first flat part corresponding to \(0 < x < g\), the second linear-like part corresponding to \(g \leq x \leq L\) and the third flat part corresponding to \(L < x < L + g\).

When \(g \leq x \leq L\), the total capacitance (Fig. 4c) consists of the capacitances between electrodes via surfaces \(a\) (\(C_{a1}\) and \(C_{a2}\)), \(b\) (\(C_{b1}\) and \(C_{b2}\)), and \(c\) (\(C_{c1}\) and \(C_{c2}\)), and the direct capacitance between electrodes (\(C_{12d}\)). Traditionally, only \(C_{a1}\) and \(C_{c2}\) are considered, which can be given by the following equations.

\[
\begin{align}
C_{a1(x)} &= \frac{\varepsilon_0 w(L - x)}{d} \quad (15a) \\
C_{c2(x)} &= \frac{\varepsilon_0 w(x - g)}{d} \quad (15b)
\end{align}
\]

Similar to eqn (6b) of the LS TENG, \(C_{a2}\) and \(C_{c1}\) are quite small, which can be approximated as small constants. Due to the structural symmetry, \(C_{b1}\) is equal to \(C_{b2}\), thus \(Y_{b} = C_{b2}/C_{b1} = 1\). Therefore, according to the charge distribution equation, eqn (3), the charge distributed on electrode 1 can be defined as:

\[
q_{1(x)} = q_{a1} + q_{b1} + q_{c1} = \frac{1}{1 + \frac{C_{a2}}{C_{a1}}} Q_{a} + \frac{1}{1 + \frac{C_{b2}}{C_{b1}}} Q_{b} + \frac{1}{1 + \frac{C_{c2}}{C_{c1}}} Q_{c}
\]

\[(g \leq x \leq L)\]

(16)

When the distance \(d\) is fixed, \(Y_{b}\), \(Z_{a}\) and \(Z_{c}\) are considered as constants, for convenience of the calculation:
Therefore, the charge distributed on electrode 1 can be defined as:

$$q_{1(x)} = \sigma w \left( \frac{L - Z_a}{1 + Y_b} + Z_c \right) - \sigma w \left( x - \frac{Z_a^2}{(L - x)} + \frac{Z_c^2}{(x - g) + Z_c} \right) \quad (g \leq x \leq L) \quad (18)$$

With $x = 0$ being assigned to the reference state, the short-circuit charge $Q_{sc}$ can be calculated using the equation below.

$$Q_{sc(x)} = |q_{1(0)} - q_{1(x)}| = \sigma w \left( \frac{Z_a^2}{L + Z_a} - \frac{g}{1 + Y_b} - Z_c \right) + \sigma w \left( x - \frac{Z_a^2}{(L - x) + Z_a} + \frac{Z_c^2}{(x - g) + Z_c} \right) \quad (19)$$

It should be emphasized that the second part of $Q_{sc}$ does not have a strict linear relationship with displacement. Because the constants $Z_a$ and $Z_c$ are quite small, the items $Z_a^2/(L - x + Z_a)$ and $Z_c^2/(x - g + Z_c)$ do not have a significant influence on $Q_{sc}$, indicating that $Q_{sc}$ seems to have a linear relationship with displacement when $g \leq x \leq L$.

When the distance $d = 0$, the $Z_a$ and $Z_c$ are zero and $Q_{sc}$ can be simplified as.

$$Q_{sc(x)} = \sigma w(x - g/2) \quad (20)$$

This equation demonstrates that when the distance $d$ is zero, Part 2 of $Q_{sc}$ has a linear relationship with displacement $x$.

When $0 < x < g$, the total capacitance consists of the capacitances between electrodes via surfaces $a$ ($C_{a1}$ and $C_{a2}$) and $b$ ($C_{b1}$ and $C_{b2}$), and the direct capacitance between electrodes ($C_{12d}$). Fig. 4f shows that the $Q_{sc}$ of Part 1, the framed part in Fig. 4e, increases slowly at the beginning and then much faster. The EDAEC method mentioned above is used to quantitatively describe the non-linear relationship of $Q_{sc}$. The equivalent capacitance of an SFT TENG when $0 < x < g$ is shown in Fig. 4b.

Due to the edge capacitance $C_{bd1}$ or $C_{bd2}$ between surface b or a of the dielectric material and electrode 2, quite a bit of charges will be induced to distribute on electrode 2, causing the slow non-linear increase. According to the charge distribution equation, eqn (3), as $x$ increases, the ratio of $C_{bd1}/C_{bd2}$ will be bigger.
causing more and more charges to be transferred to electrode 2. This non-linear increase phenomenon could not be quantitatively explained without considering the edge capacitance. The charge distributed on electrode 1 could be given as:

\[ q_{1(s)} = q_{a1} + q_{b1} = \frac{1}{1 + \frac{C_{a1}}{C_{sc}}} Q_a + \frac{1}{1 + \frac{C_{b1}}{C_{sc}}} Q_b \]

\[ = \sigma_w(L - Z_a) - \sigma_w x \left( 1 - \frac{1}{1 + Y_b} \right) + \frac{\sigma_w Z_a^2}{(L - x) + Z_a} \]

\[ (0 < x < g) \] (21)

The \( Q_{sc} \) for \( L < x \leq L + g \) (Part 3) can be directly given due to its symmetry with that for \( 0 \leq x \leq g \) (Part 1), and then the charge distributed on electrode 1 could be given as:

\[ q_{1(s)} = \begin{cases} \sigma_w(L - Z_a) - \sigma_w x \left( 1 - \frac{1}{1 + Y_b} \right) + \frac{\sigma_w Z_a^2}{(L - x) + Z_a} & 0 \leq x < g \\ \sigma_w(L - Z_a + \frac{g}{2} + Z_c) - \sigma_w x \left( 1 - \frac{Z_a^2}{(L - x) + Z_a} + \frac{Z_c^2}{(x - g) + Z_c} \right) & g \leq x \leq L \\ \sigma_w\left( L + \frac{g}{2} + Z_c \right) - \sigma_w x \left( 1 + \frac{1}{Y_b} \right) - \frac{\sigma_w Z_c^2}{(x - g) + Z_c} & L < x \leq L + g \end{cases} \] (22)

Thus, the short-circuit charge \( Q_{sc} \) can be calculated using the equation below:

\[ Q_{sc(s)} = \begin{cases} \frac{\sigma_w Z_a^2}{L + Z_a} + \sigma_w x \left( 1 - \frac{1}{1 + Y_b} \right) - \frac{\sigma_w Z_a^2}{(L - x) + Z_a} & 0 \leq x < g \\ \sigma_w\left( L - \frac{Z_a^2}{L + Z_a} - \frac{g}{2} - Z_c \right) + \sigma_w x \left( 1 - \frac{Z_a^2}{(L - x) + Z_a} + \frac{Z_c^2}{(x - g) + Z_c} \right) & g \leq x \leq L \\ \sigma_w\left( L + \frac{Z_a^2}{L + Z_a} - \frac{L + g}{1 + Y_b} - Z_a - Z_c \right) + \sigma_w x \left( 1 + \frac{1}{Y_b} \right) + \frac{\sigma_w Z_c^2}{(x - g) + Z_c} & L < x \leq L + g \end{cases} \] (23)

In order to validate the above theoretical calculation, an SFT TENG model is simulated using COMSOL, which also takes the edge effect into consideration, and the corresponding results are plotted in Fig. 4e and f. It should be emphasized that the \( Q_{sc} \) from eqn (23) is consistent with the simulated \( Q_{sc} \) by COMSOL.

When the distance \( d = 0 \), \( Z_a \) and \( Z_c \) are zero and \( Q_{sc} \) can be simplified as:

\[ Q_{sc(s)} = \begin{cases} \sigma_w x \left( 1 - \frac{1}{1 + Y_b} \right) & 0 \leq x < g \\ \sigma_w x - \sigma_w g/2 & g \leq x \leq L \\ \sigma_w\left( L - \frac{L + g}{1 + Y_b} \right) + \sigma_w x \left( 1 + \frac{1}{1 + Y_b} \right) & L < x \leq L + g \end{cases} \] (24)

The total capacitance \( C_{total} \) and the short-circuit transferred charge \( Q_{sc} \) have already been quantitatively given, and thus the open-circuit voltage can be obtained using eqn (14). The voltage is also divided into three parts according to \( x \), as shown in Fig. 4g. Due to the edge capacitance, there is a slow-increase part of Part 1 (\( 0 < x < g \)) as shown in Fig. 4h. The voltage of Part 2 (\( g \leq x \leq L \)) has a linear-like relationship with \( x \). Due to the increasing edge capacitance effect, the increase of the voltage of Part 3 (\( L < x < L + g \)) will turn flat again.

According to eqn (23), the short-circuit transferred charge \( Q_{sc} \) is also affected by the TENG’s structural parameters, for example the distance \( d \) between the dielectric material and the electrodes, and the gap \( g \) between the two electrodes. Therefore, the transferred charge and open-circuit voltage with different \( d \) and gap \( g \) values are also shown in Fig. 4i–l. The slopes of \( Q_{sc} \) and \( V_{oc} \) become a little smaller when the distance \( d \) increases, as shown in Fig. 4i and j. Compared with the influence of \( d \) on the LS TENG in Fig. 3c and d, it seems that the distance \( d \) has little influence on the transferred charge and open-circuit voltage, which is consistent with experimental results of Wang et al. on an SFT TENG with a certain distance \( d \), to significantly extend the lifetime.28

When the gap \( g \) becomes larger, the ratio of Part 1 and Part 3 becomes larger. And thus the influence of edge capacitance becomes more significant. As a result, the non-linear part becomes more obvious, and the flat parts of \( Q_{sc} \) at both ends are much more noticeable. Besides, the total capacitance will decrease due to the increase of gap \( g \). The total transferred charge remains the same during the whole process, and therefore the open-circuit voltage will increase, as shown in Fig. 4I. Meanwhile, the portions of Part 1 and 3 also become larger. The results simulated by COMSOL are also presented, which are well-consistent. Based on the above analysis, the new method shows high accuracy in describing the short-circuit charge \( Q_{sc} \) and open-circuit voltage \( V_{oc} \) of SFT TENGs.

SE sliding mode

Owing to only one electrode, it is difficult for researchers to quantitatively describe the output performances of SE sliding (SES) TENGs.28 Meanwhile some researchers reported the
output performances of SES TENGs, especially the transferred charge and the energy conversion efficiency for applications. Therefore, a quantified mode is strongly required to describe SES TENGs’ special performance. The EDAEC method mentioned above can solve this problem. The mode in Fig. 5a is designed to demonstrate the working principle of an SES TENG, as the ground could be regarded as the second electrode of a single electrode TENG. Similar to the method described before, metal electrode 1 and the dielectric layer are defined with length $L = 100$ mm and width $w = 100$ mm. The distance

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**Fig. 5** The schematic (a) and equivalent capacitance (b) of the SES TENG. (c) The relationship of transferred charge with displacement. (d) The relationship of open-circuit voltage with displacement. (e and f) The relationship of transferred charge at short-circuit (e) and open-circuit voltage (f) with the distance between the dielectric layer and the ground $d_g$. (g) The similar structure of the SES TENG (when $d_g$ is small) and LS TENG. (h) The similar structure of the SES TENG (when $d_g$ is large) and SFT TENG (Part 1).
between the dielectric layer and electrode 1 is defined as \( d \), and the distance between the dielectric layer and the ground is defined as \( d_g \). The surrounding environment connected to the ground will have an effect on \( d_g \), which could be regarded as a big conductive box containing the SES TENG.

The equivalent capacitance of \( C _ { \text { total} } \) is shown in Fig. 5b, which includes three parts, including the capacitances between electrode 1 and the ground electrode via surfaces \( a \) (\( C _ { a1 } \) and \( C _ { a2 } \)), and \( b \) (\( C _ { b1 } \) and \( C _ { b2 } \)). Because the ground electrode surrounds the dielectric layer, there are several in-parallel capacitances \( C _ { bg,n } \) between the dielectric layer and each surrounding surface, and \( C _ { bg } \) is the total capacitance. And \( C _ { 1gd } \) is the direct capacitance between electrode 1 and the ground electrode. The capacitance can be defined using the following equations.

\[
\begin{align}
C _ { a1(s) } &= \frac{\varepsilon _ 0 w (L - x)}{d} \quad (25a) \\
C _ { a2(s) } &= \frac{\varepsilon _ 0 w (L - x)}{d_g} \quad (25b) \\
C _ { bg(s) } &= \frac{\varepsilon _ 0 w (x)}{d_g} \quad (25c) \\
\frac{1}{d_g} &= \sum ^ n _ { i=1} \frac{1}{d _ { i,g}} \quad (25d)
\end{align}
\]

According to the EDAEC method we described before, the edge capacitance \( C _ { b1 } \) is quite small compared with other capacitances. Therefore, \( C _ { b1 } \) can be approximated as a small constant. The total capacitance could be defined as:

\[
C _ { \text { total} } = C _ { 1ag } + C _ { 1gd } + C _ { 1bg } = \frac{C _ { a1 } \times C _ { a2 } + C _ { 1gd } + C _ { b1 } \times C _ { bg }}{C _ { a1 } + C _ { a2 } + C _ { 1gd } + C _ { b1 } + C _ { bg }} \quad (26)
\]

After contact electrification, the upper surface of the dielectric layer would be filled with triboelectric charges. Under short-circuit conditions, the total charge on electrode 1 can be calculated using the following equation.

\[
q _ { 1(s) } = q _ { 1ag } + q _ { 1gd } + q _ { 1bg } = \frac{1}{1 + \frac{C _ { a2 } }{C _ { a1 } } + \frac{C _ { bg } }{C _ { b1 } } + \frac{C _ { 1gd } + C _ { b1 } \times C _ { bg } }{C _ { a1 } + C _ { a2 } + C _ { 1gd } + C _ { b1 } + C _ { bg }}} \quad (27)
\]

In the equation above, to facilitate the calculation, the constant is defined as:

\[
Z = \frac{C _ { b1 } d _ g}{\varepsilon _ 0 w} \quad (28)
\]

Thus, the short-circuit charge \( Q _ { \text { sc } } \) can be calculated using the equation below.

\[
Q _ { \text { sc(s) } } = |q _ { 1(0) } - q _ { 1(s) } | = \left( \frac{d _ g + d }{d _ g + d + d} \right) \sigma w x + \frac{1}{Z + x} - Z \sigma w \quad (29)
\]

Due to the characteristics of single electrode TENGs, the short-circuit charge \( Q _ { \text { sc } } \) is noticeably lower than the total charge on electrode 1, \( Q _ { \text { total } } \), which is equal to the total triboelectric charge of the dielectric layer. From Fig. 5c, at \( x = L _ s \), the short-circuit charge \( Q _ { \text { sc } } \) only reaches around 70% of the total movable charge \( Q _ { \text { total } } \). This phenomenon could explain why the output power or the efficiency of SE TENGs is quite lower than that of LS or SFT TENGs. To validate the above equations, an SES TENG model is simulated by COMSOL. The transferred charge from the equation and the numerical results simulated from COMSOL are plotted in Fig. 5c which are quite consistent with our model.

With the total capacitance \( C _ { \text { total } } \) and the short-circuit charge \( Q _ { \text { sc } } \) known, the open-circuit voltage can be obtained using eqn (14). The open-circuit voltage from the equation and the results simulated from COMSOL are plotted on the same graph for comparison in Fig. 5d. The theoretical calculation coincides with numerical simulation, which demonstrates the accuracy of the proposed theoretical model.

According to eqn (29), the distance \( d _ g \) between the dielectric material and the ground electrode will significantly affect the transferred charge \( Q _ { \text { sc } } \). Therefore, the transferred charge and open-circuit voltage with different distance \( d _ g \) values are also shown in Fig. 5e and f. It is interesting to notice that the slope of \( Q _ { \text { sc } } \) is much smaller when the distance \( d _ g \) increases, while that of \( V _ { \text { sc } } \) is much larger. When \( d _ g \) is smaller, the ratio of \( Q _ { \text { sc } } / Q _ { \text { total } } \) of the SES TENG is bigger and the shape of \( Q _ { \text { sc } } \) is very similar to the linear-like \( Q _ { \text { sc } } \) of the LS TENG, due to the structural similarity as shown in Fig. 5g. Meanwhile, the voltage of the LS TENG (Fig. 3b) is much lower compared with the voltage of the SFT TENG (Fig. 4g). Interestingly, when \( d _ g \) is small, the open-circuit voltage of the SES TENG is much smaller, and the shape and the value of the voltage are similar to those of the LS TENG.

When \( d _ g \) is larger, the ratio of \( Q _ { \text { sc } } / Q _ { \text { total } } \) of the SE TENG is smaller and the shape of \( Q _ { \text { sc } } \) is very similar to that of the non-linear \( Q _ { \text { sc } } \) of the SFT TENG (Part 1: \( 0 < x < g \)). As shown in Fig. 5b, when the gap \( g \) between the electrodes is significantly larger than the length of the electrode, the structures of the SES TENG and the SFT TENG become similar, and thus they have a similar \( Q _ { \text { sc } } \) shape. Therefore, with greater \( d _ g \), the flattening in the small sliding distance would be more obvious (Fig. 5e). Meanwhile, the shape and the value of the open-circuit voltage of the SES TENG become very similar to those of the sliding FT TENG. It should be concluded that the outputs of the SES TENG are similar to those of the LS TENG when \( d _ g \) is small, while they are similar to those of the SFT TENG when \( d _ g \) is large.

**Conclusion**

A universal EDAEC (edge approximation based equivalent capacitance) method is built to establish precise analytical models for all kinds of TENGs, in which the equivalent capacitance models and the side effect are two key points. The equivalent capacitance models are applied to demonstrate charge distributions on each electrode. And the capacitances of edge effect are estimated. Therefore, quantitative analysis formulae for TENGs from the universal method and the
quantitative $Q-V-x$ relationship are established. Although this research mainly focused on sliding TENGs, the universal method can also be applied for contact-separation TENGs. Meanwhile, this method can also be used for complicated structures of TENGs in the future. Therefore, a universal method, which can give the quantitative analysis formulae (the quantitative $Q-V-x$ relationship) for all kinds of TENGs becomes more and more important. Finally, the simulated results based on the finite element method (COMSOL software) are provided to make a comparison with the quantitative analysis from the universal EDAEC method, and they are in good agreement. A universal method for all kinds of TENGs is a milestone to provide more delicate analytical models for in-depth understanding of the working principles of different TENGs.

Conflicts of interest

There are no conflicts to declare.

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